**Lecture**

**WEEK #13**

**Chapter 10 Graphs and Trees**

Section 10.1 Graphs: An Introduction

Become familiar with the definition of a **graph: two finite sets of vertices and edges with each edge is associated with one or two vertices** and the vocabulary needed: vertex, vertices, edges, loop, parallel edges, isolated vertex, adjacent, incident on, endpoints, edge-endpoint tables.

Special graphs begin on p.632.

**Simple graphs** have no loops or parallel edges.

A **complete graph** is a simple graph that has exactly one edge for each pair of distinct vertices

A complete bipartite graph appears on p.633 along with the definition of a subgraph.

The concept of **degree of a vertex** is the count of the number of edges going in or coming out of that vertex. If there is a loop, it goes into the vertex and out of the vertex so it adds 2 to the degree count.

Several properties of graphs are shown in theorems and corollaries:

1. The total degree of a graph is = sum of the degrees at each of its vertices.
2. The total degree of a graph = twice the number of edges.
3. The total degree of a graph is even
4. In any graph there are an even number of vertices of odd degree.

Read through the proofs and walk through the worked out examples and applications as in the ‘acquaintance graph’ p.637 and **use** the definitions and the properties.

Section 10.2 Trails, Paths and Circuits.

This section starts with the famous Konigsberg Bridge Problem. You can see how graph theory can be applied to real life situations.

On page 644 you expand the definition of graph, each definition adds one additional condition to the previous definition:

Walk: finite alternating sequence of adjacent vertices and edges

Trail: from v to w is a walk that does not contain a repeated edge

Path; from v to w is a path that does not contain a repeated vertex [remember the path also does not contain a repeated edge from previous definition]

Closed Walk: a walk that starts and ends in the same vertex

Circuit: is a closed walk that does not repeat an edge

Simple Circuit: is a circuit that does not have a repeated vertex except for first and last.

There is a nice summary chart on the top of page 645.

Additionally:

Connected Graph: there is a walk from any two vertices

Euler Circuit: circuit that contains every vertex and every edge – since it is a circuit it cannot repeat an edge and since a circuit is a closed walk, it must start and end in the same vertex.

**Carefully read through the worked out Examples 10.2.1 and 10.2.2 and 10.2.3**

Read Theorem 10.2.2 on page 648 about Euler circuits. If a graph has an *Euler circuit*, then every vertex of the graph has **even** degree and the **contrapositive**: If some vertex of the graph is odd, it does not have an Euler circuit is useful when looking for circuits in diagrams.

Now an *Euler Path* is a path [can’t repeat an edge] from v to w in a sequence of adjacent vertices passing through every vertex at least once and traversing every edge exactly once.

There is a child’s game where you have to trace the diagram without taking your pencil off the paper and you can only trace a line once. See if you can find the Euler path in the diagram below. You will see that you must **start** and **end** in the **odd degree vertices**.

This puzzle is solved using Corollary 10.2.5 on page 653 and the two bottom vertices which are the ***only*** odd ones in the drawing must be where you begin and the other where you end.

Any questions?